Calculation of Correlation Coefficient in CIPT fitting result

Introduction

The CIPT result sheet includes two indicators of the quality of the fit, the fitting parameter "*f*" and the *Correlation Coefficient* (*cc*).

This Application Note describes how the *Correlation Coefficient* (*cc*) is calculated. The *cc* provides a figure of how well the measured $R_{sq,low}$-values (shown as red crosses "✕") correlate to the fitted $R_{sq,low}$-values (shown as blue circles "○").

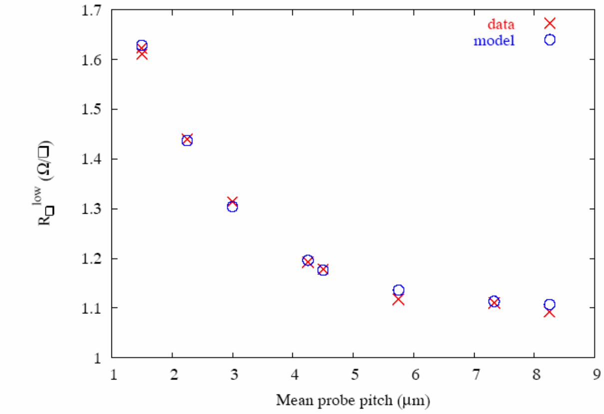

Figure 1 Example of measured (✕ - "data") and fitted (○ - "model") $R_{sq,low}$ vs. Mean probe pitch.

Details about the Correlation Coefficient can be found e.g. at "MathWorld"'s homepage, http://mathworld.wolfram.com/CorrelationCoefficient.html

Calculation of cc

The following denotations are used:

x_i: Measured $R_{sq,low}$ for the *i*-th pin-configuration ($i \in [1; N]$, where N is the total number of pin-configurations used)

$\bar{x}$ Average (mean) value of the N measured $R_{sq,low}$-values

y_i Fitted $R_{sq,low}$ for the *i*-th pin-configuration ($i \in [1; N]$, where N is the total number of pin-configurations used)

$\bar{y}$ Average (mean) value of the N fitted $R_{sq,low}$-values

The *sums of squared values* (*ss*) are then calculated:

$$ss_{xx} = \sum (x_i - \bar{x})^2$$

$$ss_{yy} = \sum (y_i - \bar{y})^2$$

$$ss_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y})$$

The *Correlation Coefficient cc* is then defined as

$$cc^2 = \frac{ss_{xy}^2}{ss_{xx} ss_{yy}}$$

Example

Consider the following 2 sets of measured and fitted values:

$\{R_{sq,low(measured)}\} = \{x\} =$
$\{1.08761;1.09815;0.932981;0.890418;0.876494;0.881817;0.878098;0.883042;0.870126\}$

$\{R_{sq,low(fitted)}\} = \{y\} =$
$\{1.08761;1.08761;0.985074;0.92593;0.88842;0.882247;0.8724;0.868044;0.867185\}$

Then

$$\bar{x} = 0.93319$$

$$\bar{y} = 0.94050$$

$$ss_{xx} = \sum_{i=1}^{9}\left(x_i - \bar{x}\right)^2 = 0.068267$$

$$ss_{yy} = \sum_{i=1}^{9}\left(y_i - \bar{y}\right)^2 = 0.066850$$

$$ss_{xy} = \sum_{i=1}^{9}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) = 0.065552$$

$$\Downarrow$$

$$cc^2 = 0.941578$$

$$\Downarrow$$

$$cc = 0.97035$$